DETERMINATION OF THE CARRYING CAPACITY OF GROUND ANCHORS WITH THE CORRELATION AND REGRESSION ANALYSIS

Calcul de la force portante des tirants d'ancrage à l'aide de l'analyse de la corrélation et de la régression

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SOMMAIRE

Au cours de la dernière décennie, on a effectué un grand nombre d'essais de tirants d'ancrage à Hannovre (Allemagne Fédérale). La communication décrit l'interprétation des observations recueillies par application des méthodes statistiques et propose deux équations pour la capacité portante des tirants d'ancrage : l'une pour les sols pulvérulents et l'autre pour les sols cohérents.

SUMMARY

In the last decade a great number of fundamental and suitability tests of ground anchors have been performed in Hannover, GRF. These tests have been supervised by the «Institut für Grundbau und Bodenmechanik, Technische Universität Hannover», which also performed the laboratory soil investigations . This contribution describes the evaluation of the gathered observations by application of statistical methods. The result of these investigations is the proposal of two equations to estimate the carrying capacity of ground anchors, one for anchors in non-cohesive soil and the other for anchors in cohesive soil.

INTRODUCTION

All equations estimating the carrying capacity of ground anchors published till now have been developed on the basis of theoretical considerations or model tests. Knowing these equations and taking into consideration the experience gained at site, it must be stated that the carrying capacity depends on a multitude of influence-factors, which can be classified into the following groups:

- dimensions and construction specification of ground anchors;
- soil characteristics;
- state of stress acting on the grouted body of ground anchors.

It can be supposed that there is no functional relation between the carrying capacity and each of the influence-factors, but only a stochastic relation. In natural science and in many branches of technology correlation and regression analysis are used to examine stochastic relations. Regression is a method for getting the best formula relating a dependent variable to independent variables and to determine the constants in this formula by minimizing the sum of squared deviations of observations from the regression equation. The regression equation can be a straight line (linear regression) or a curved line (nonlinear regression) and if there are more than one independent variable (simple regression) it is called multiple regression.

The interdependence between the variables is measured by the correlation coefficient. The simple correlation coefficient shows how much two variables are associated. It varies between + 1.0 (perfect correlation) through 0 (no association) to — 1.0 (perfect inverse correlation). The multiple correlation coefficient is a measurement for the association between a dependent variable and two or more independent variables. It varies between 0 (no correlation) to 1.0 (perfect correlation). The partial correlation coefficient measures the association between a dependent variable and only one independent variable, when the other independent variables are held constant. It is also measured in the range + 1.0 to — 1.0.

LIST OF SYMBOLS

The following notations are used in this paper:

a_i : regression constants.

A : carrying capacity of ground anchors in kN.

B : relative importance factor.

c' : cohesion of cohesive soil in kN/m².

$C_c = d_{30}^2/d_{10} \cdot d_{60}$: coefficient of curvature.

d_A : diameter of the grouted body.

d_w : effective diameter in mm.

d_{10} : soil diameter at which 10% of the soil weight is finer.

d_{30} : soil diameter at which 30% of the soil weight is finer.

d_{60} : soil diameter at which 60% of the soil weight is finer.

D_1 : percentage of soil with grain diameters $d < 0.006$ mm.

D_2 : percentage of soil with grain diameters $0.006 \text{ mm} < d < 0.02$ mm.

D_3 : percentage of soil with grain diameters $0.02 \text{ mm} < d < 0.06$ mm.

D_4 : percentage of soil with grain diameters $d > 0.06$ mm.

D_5 : percentage of soil with grain diameters $d < 0.2$ mm.

D_6 : percentage of soil with grain diameters $0.2 \text{ mm} < d < 0.6$ mm.

D_7 : percentage of soil with grain diameters $0.6 \text{ mm} < d < 2.0$ mm.

D_8 : percentage of soil with grain diameters $d > 2.0$ m.

F_M : superficies of the bond-to-ground length in m^2.

h_m : height of the overburden at the half of the bond-to-ground length in m.

I_c : consistency index.

I_p : plasticity index

k : coefficient of permeability.

l_o : bond-to-ground length in m.

p : maximum injection pressure.

$U = d_{60}/d_{10}$: uniformity coefficient

V_{ca} : lime content in percent.

w : natural water content in percent.

w_L : liquid limit in percent.

w_p : plastic limit in percent

α : angle of inclination of ground anchors in grade.

γ : unit weigth in kN/m^3.

φ' : angle of friction in grade.

τ : shear stress in non-cohesive soil in kN/m^2.

τ_c : shear stress in cohesive soil in kN/m^2.

INFLUENCES ON THE CARRYING CAPACITY

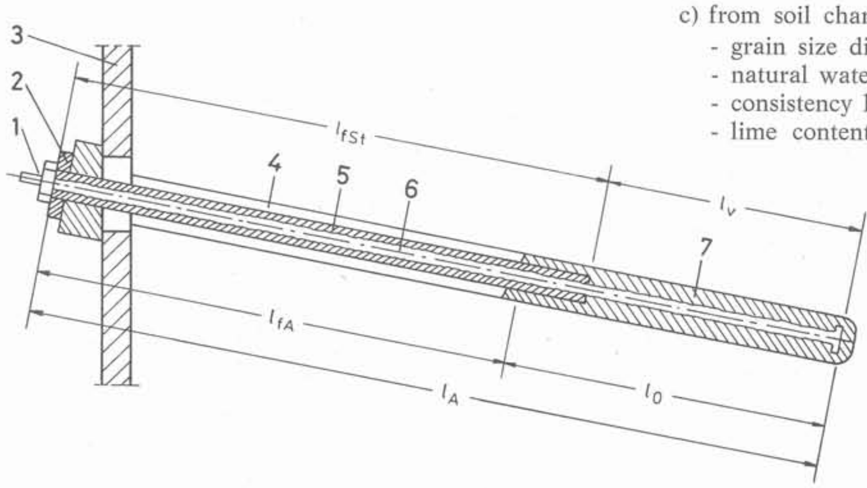
The results of about 130 fundamental and suitability tests were the basis of the following investigations. Only in some cases could the breaking load of the tested ground anchors be measured and therefore it was necessary to estimate the maximum load. By evaluating the stress-strain curves from the tests as proposed by T.F. Herbst (1971) and H. Grade (1974) the real placing bond-to-ground length could be found out. Using these determined bond-to-ground lengths the following maximum loads could be estimated: for anchors in non-cohesive soil according to H. Grade (1974) in the range of 614 kN to 1167 kN and for the anchors in cohesive soil as proposed by F.R. Hahn (1974) in the range of 181 kN to 954 kN.

The influence of the following factors on the carrying capacity was taken into consideration:

a) from the dimensions and construction specification of the anchors:
   - bond-to-ground length;
   - superficies of the bond-to-ground length;
   - inclination of the anchors;
   - maximum injection pressure;

b) from soil characteristics of non-cohesive soil:
   - grain size distribution;
   - uniformity coefficient;
   - effective diameter;
   - coefficient of permeability;

c) from soil characteristics of cohesive soil:
   - grain size distribution;
   - natural water content;
   - consistency limits;
   - lime content;



1 Anchor head
2 Bearing plate
3 Wall
4 Borehole
5 Protective sheath
6 Steel tendon
7 Grouted body

l_{fSt} Free tendon length
l_v Bond length of tendon
l_{fA} Free anchor length
l_0 Bond-to-ground length
l_A Total anchor length

Fig. 1. — **Temporary anchor construction of the investigated typ.**

d) from the state of stress acting on the grouted body:

- normal stress;
- shear stress;

both estimated by different formulas.

The aim of these investigations was to find answers to the following questions:

1) Which factors can have an influence on the carrying capacity?
2) How intense is the dependence of the carrying capacity on individual factors?
3) What are the direct associations between the carrying capacity and the influencing factors?

This paper deals only with temporary anchors as shown in fig. 1.

CORRELATION- AND REGRESSION ANALYSIS OF GROUND ANCHORS IN NON-COHESIVE SOIL

The test results of 51 ground anchors were taken into consideration. Using the linear multiple regression and taking 8 influence-factors in each case multiple correlation coefficients between 0.83 and 0.97 were measured. The formula yielding the highest value included the following 8 independent variables:

$$A = f\,(F_M, p, D_5, D_6, D_7, D_8, k, \tau) \qquad (1)$$

To determine the degree of association between the carrying capacity and only one of the 8 influence-factors of formula (1), partial correlation coefficients have been computed.

Comparatively the superficies of the bond-to-ground length F_M has been exchanged for the bond-to-ground length l_o. The results are shown in the following table:

1	2	3	4	5
No	multiple correlation coefficient	investigated variables	simple correlation coefficient	partial correlation coefficient
1	0.972	A, F_M	0.471	0.543
2	0.928	A, l_o	0.350	0.459
3	0.972	A, p	— 0.263	0.261
4	0.972	A, D_5	0.239	0.000
5	0.972	A, D_6	0.039	0.000
6	0.972	A, D_7	— 0.269	0.000
7	0.972	A, D_8	— 0.175	0.000
8	0.972	A, k	0.015	— 0.789
9	0.972	A, τ	0.758	0.932

To known the influence of other soil characteristics the coefficient of permeability in the formula (1) has been exchanged for other factors. The following correlation coefficients have been computed:

1	2	3	4	5
No	multiple correlation coefficient	investigated variables	simple correlation coefficient	partial correlation coefficient
1	0.884	A, U	— 0.516	0.501
2	0.914	A, C_c	— 0.049	— 0.422
3	0.939	A, d_w	0.055	— 0.748

By using the regression analysis, the constants of the formula (1) were determined. But these constants do not adequately reveal the relative effect of the independent variables on the dependent variable because of differences in the units. The importance of the independent variables can be measured by the relative importance-factor of variables B. It can be stated that the dependent variable is influenced more by the independent variable with a larger relative important factor.

The following table shows the relative importance-factors of the variables in formula (1):

$$B\,(F_M) = 0.0000138010$$
$$B\,(p) = 0.0000000031$$
$$B\,(D_5) = 1046367.8310152777$$
$$B\,(D_6) = 1107823.6971716285$$
$$B\,(D_7) = 667740.4196914956$$
$$B\,(D_8) = 1163158.8443476185$$
$$B\,(k) = -\,0.5683856830$$
$$B\,(\tau) = 0.0000141248$$

This investigation shows that the whole grain size curve is very important for the carrying capacity, while the separate grain size groups have only a negligible influence.

As a result of a great number of calculations, the following equation to estimate the carrying capacity of ground anchors in non-cohesive soil is proposed:

$$A = a_o + a_1 \,.\, F_M + a_2 \,.\, D_5 + a_3 \,.\, D_6 + a_4 \,.\, D_7 + a_5 \,.\, D_8 + a_6 \,.\, k + a_7 \,.\, \tau \qquad (2)$$

with

$$F_M = \pi \,.\, d_A \,.\, l_o$$

and

$$\tau = \frac{2 - \sin \varphi'}{2} \,.\, \gamma \,.\, h_m \,.\, \tan \varphi'$$

The correlation analysis yielded a multiple correlation coefficient equals 0.96 and the following values for the constants of formula (2):

$$a_o = -\,2\,679.36$$
$$a_1 = +\ 34.12$$
$$a_2 = +\ 29.20$$
$$a_3 = +\ 30.94$$
$$a_4 = +\ 20.63$$
$$a_5 = +\ 31.92$$
$$a_6 = -\,2\,051.48$$
$$a_7 = +\ 9.73$$

Using formula (2) to estimate the carrying capacity it must be observed that the grain size curve lies within the boundaries of fig. 2 and the values of the influence-factors do not exceed the following limits:

$0.98\ m^2$	$\leq F_M \leq\ 3.61\ m^2$	0 %	$\leq D_7 \leq 17$ %
7.40 cm	$\leq d_A \leq 11.50$ cm	0 %	$\leq D_8 \leq 77$ %
4.10 m	$\leq l_o \leq 15.00$ m	$0.122 \cdot 10^{-2}$ cm/s	$\leq k \leq 25.2 \cdot 10^{-2}$ cm/s
0 %	$\leq D_5 \leq 86$ %	$31.7\ kN/m^2$	$\leq \tau \leq 95.6\ kN/m^2$
10 %	$\leq D_6 \leq 78$ %		

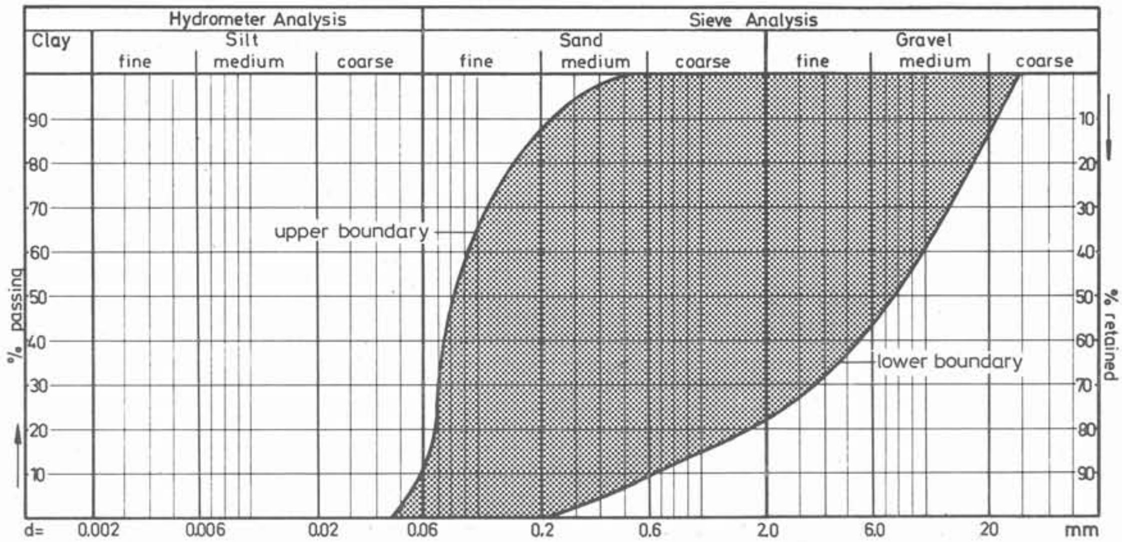


Fig. 2. — **Boundaries of the grain size distribution of the investigated non-cohesive soil.**

CORRELATION- AND REGRESSION ANALYSIS OF GROUND ANCHORS IN COHESIVE SOIL

The test results of 57 ground anchors have been investigated. Using the linear multiple regression and taking 9 influence-factors into consideration in each case multiple correlation coefficients between 0.97 and 0.98 have been computed. The formula (3) yielded the highest value.

$$A = f(F_M, p, D_1, D_2, D_3, D_4, I_c, V_{ca}, \tau_c) \quad (3)$$

The following table contains the multiple, simple and partial correlation coefficients of the above formula, in which the superficies of the bond-to-ground length F_M has been exchanged again for the bond-to-ground length l_o:

1	2	3	4	5
No	multiple correlation coefficient	investigated variables	simple correlation coefficient	partial correlation coefficient
1	0.983	A, F_M	0.508	0.752
2	0.983	A, l_o	0.057	0.569
3	0.983	A, p	0.340	0.034
4	0.983	A, D_1	— 0.698	— 0.157
5	0.983	A, D_2	— 0.021	— 0.170
6	0.983	A, D_3	0.669	— 0.210
7	0.983	A, D_4	0.800	0.385
8	0.983	A, I_c	0.638	0.093
9	0.983	A, V_{ca}	— 0.141	— 0.119
10	0.983	A, τ_c	0.676	0.666

Exchanging the consistency index I_c in the formula (3), the influence of other soil characteristics on the carrying capacity has been computed as follows:

1	2	3	4	5
No	multiple correlation coefficient	investigated variables	simple correlation coefficient	partial correlation coefficient
1	0.983	A, w	— 0.099	— 0.063
2	0.983	A, w_L	0.155	— 0.052
3	0.983	A, w_p	0.466	0.066
4	0.983	A, I_p	— 0.293	— 0.056

Subsequently the constants of formula (3) and the relative importance factors of the independent variables have been computed as already described above. The results indicate that the carrying capacity is most influenced by the superficies of the bond-to-ground length and by the consistency index.

Taking the results of these investigations as a basis, it can be proposed to estimate the carrying capacity of ground anchors in cohesive soil by the following equation:

$$A = a_o + a_1 \cdot F_M + a_2 \cdot D_1 + a_3 \cdot D_2 + a_4 \cdot D_3 + a_5 \cdot D_4 + a_6 \cdot I_c + a_7 \cdot \tau_c \quad (4)$$

with

$$F_M = \pi \cdot d_A \cdot l_o$$

and

$$\tau_c = \frac{\gamma \cdot h_m \cdot \tan \varphi'}{\cos^2 \alpha + \sin^2 \alpha (1 + 2 \cdot \tan^2 \varphi') + 2 \cdot \sin \alpha \cdot \cos \alpha} + c' \cdot \cos^2 \varphi'$$

The multiple correlation coefficient for this equation was 0.98 and the following values for the constants have been calculated:

$$a_o = +\ 721.51$$
$$a_1 = +\ 71.84$$
$$a_2 = -\ 9.81$$
$$a_3 = -\ 1.99$$
$$a_4 = -\ 21.22$$
$$a_5 = +\ 10.34$$
$$a_6 = +\ 95.15$$
$$a_7 = +\ 2.56$$

Estimating the carrying capacity of ground anchors in cohesive soil by using formula (4), the grain size curve must be within the boundaries of fig. 3 and the values of the influence-factors are not allowed to exceed the following limits:

$$0.98\ m^2 \leq F_M \leq 6.48\ m^2$$
$$6.50\ cm \leq d_A \leq 16.80\ cm$$
$$4.10\ m \leq l_o \leq 15.00\ m$$
$$20\ \% \leq D_1 \leq 76\ \%$$
$$12\ \% \leq D_2 \leq 27\ \%$$
$$4\ \% \leq D_3 \leq 27\ \%$$
$$2\ \% \leq D_4 \leq 34\ \%$$
$$0.84 \leq I_c \leq 1.35$$
$$50.7\ kN/m^2 \leq \tau_c \leq 165.3\ kN/m^2$$

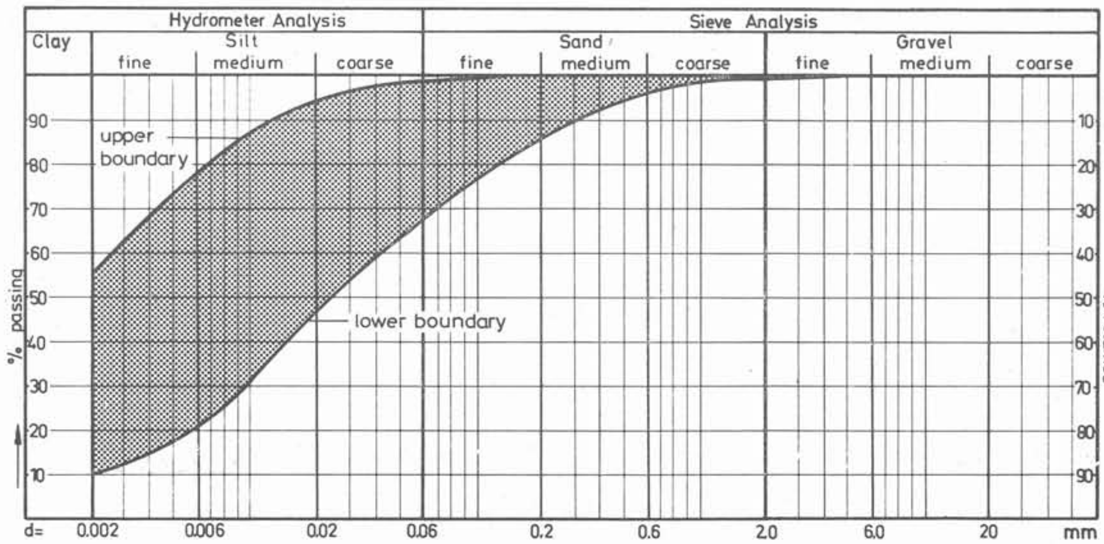


Fig. 3. — **Boundaries of the grain size distribution of the investigated cohesive soil.**

CONCLUSIONS

The evaluation of the test results of 51 ground anchors in non-cohesive as well as 57 ground anchors in cohesive soil by using the correlation and regression analysis shows the influence of ground anchor dimensions, soil characteristics and the shear stresses acting on the grouted body on the carrying capacity of ground anchors. On the basis of these investigations two equations to estimate the carrying capacity are proposed, one for ground anchors in non-cohesive soil and the other one for ground anchors in cohesive soil. The constants of these equations, including 7 parameters in each case, have been determined by means of the linear multiple regression analysis.

Both equations include the grain size distribution of the soil classified in 4 fractions, the superficies of the bond-to-ground length and the shear stress acting on the superficies of the bond-to-ground length. Another parameter in the formula for ground anchors in non-

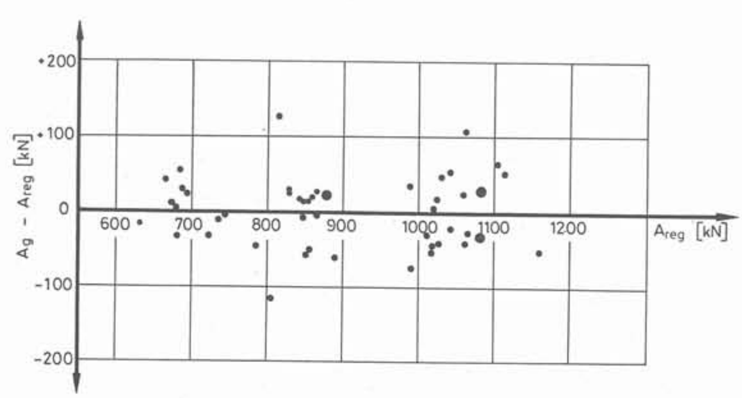
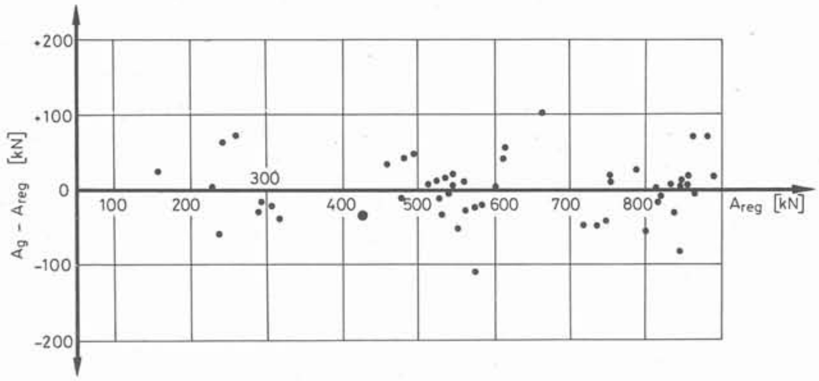


Fig. 4. — **Differences between the carrying capacity of ground anchors in non-cohesive soil gained by evaluating the stress-strain curves (A_g) and estimated by the proposed formula (A_{reg}).**

Fig. 5. — **Differences beween the carrying capacity of ground anchors in cohesive soil gained by evaluating the stress-strain curves (A_g) and estimated by the proposed formula (A_{reg}).**



cohesive soil is the coefficient of permeability, whereas in the formula for ground anchors in cohesive soil the consistency index appears. Furthermore the unit weight, the angle of friction and the cohesion of cohesive soil appear in both equations. Estimating with the proposed equations the carrying capacity of the ground anchors used in the regression analysis, it is found that about 90 percent of the ground anchors have only differences of less than 60 kN from the loads estimated by evaluating the stress-strain curves from the tests as shown in fig. 4 and 5.

In spite of the good correlation demonstrated in fig. 4 and 5 it must be stated that it is not possible to abandon the acceptance test of each ground anchor at site due to stratifaction and nonhomogeneity of soil. Furthermore there is no possibility to take the different placing methods into consideration. The advantage of the proposed equations based on performed tests at site is that reliable loads can be estimated already during the stage of design without special expenditures or time delay.

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