NON LINEAR ANALYSIS OF THE ANCHOR-GROUND-WALL SYSTEM

Analyse non linéaire de l'ensemble ancrage-sol-paroi

by

Ass. Professor Mihail POPESCU

Soil Mechanics and Foundation Engineering Chair, Civil Engineering Institute, Bucharest, Romania

and

Lecturer Crișan IONESCU

Mathematics Chair, Civil Engineering Institute, Bucharest, Romania

SOMMAIRE

Les dispositifs d'ancrages se sont montrés particulièrement économiques pour assurer le soutien des parois des excavations lorsqu'elles sont grandes. Le projet d'un mur flexible ancré est une opération complexe qui dépend de la connaissance de l'interaction entre l'ancrage, le sol et le mur. Dans la présente étude, on montre un procédé numérique qui prend en considération cette interaction.

Le premier but de l'analyse faite fut d'examiner au moyen de ce mode de calcul, la dépendance entre la force d'ancrage et un certain nombre de variables primaires comme la flexibilité du mur, la raideur de l'ancrage, la profondeur de la fiche et les conditions initiales des contraintes, variables qui régissent le comportement des murs ancrés à plusieurs niveaux. Les valeurs calculées de la réaction d'ancrages ont été comparées à celles prédites par les règles semi-empiriques usuelles.

Les solutions proposées dans cette communication sont basées sur l'analyse d'un grand nombre de situations. Elles sont utiles, à notre avis, pour l'évaluation par l'ingénieur de l'ensemble du problème du comportement d'une structure ancrée.

SUMMARY

For wide excavations the tied-back system has proved a particularly economical way of securing the walls of excavations. The design of a tied-back flexible wall is a complex operation which depends on a knowledge of the interaction between the anchors, the ground and the wall. In the present study, a practical numerical procedure taking into consideration this interaction, is introduced.

The primary purpose of the undertaken analysis was to examine, by means of this procedure, the dependence of the anchor force on some of the primary variables, such as the wall flexibility, anchor stiffness, embedment depth and initial stress conditions, controlling the performance of the multianchored walls. The computed anchor reaction values were compared with those predicted by means of the usual semi-empirical rules.

The paper findings, based upon a large number of analysed situations, are, in our opinion, useful for engineering evaluation of the over-all problem of the anchor systems behaviour.

INTRODUCTION

The tied-back wall is employed with increasing frequency as a form of support for deep temporary and permanent excavations, due to the improvement of grouted anchors which offer high bearing capacities at fairly low costs (Habib, 1969). Although the combined «anchor-ground-wall» system has been investigated extensively and the behaviour of anchors has been known for some time, the effect of various factors influencing the anchor force is not well documented.

The recent development of modern computers, which are able to handle the comprehensive programmes required to calculate the stresses and strains in a continuum with specified boundary conditions, has opened up a possibility of obtaining improved solutions to the problem of anchored flexible walls. These programmes which are based on the finite element method (Clough, Tsui, 1974) are still not developed far enough to be used for design purposes. Moreover their use is limited by the high computing cost.

In the present study, a practical numerical procedure to determine stresses and strains in an anchored flexible wall is introduced, taking into account the nonlinear soil and anchor behaviour. Comparison of the results obtained by this procedure with the finite element method results has shown good agreement.

The behaviour of single-anchored flexible wall has been studied both theoretically and experimentally by Rowe (1955). Rowe's design curves are accepted and used now, in general, by the profession.

As the use of multianchored walls developed, the empiricial designs for multistrutted walls were applied, despite the fact that there were basic differences in behaviour between the two support systems.

For this reason it was decided to carry out a parametric analysis of the multianchored walls, using the proposed procedure (Popescu, 1977). This study is limited to the anchor force dependence on some of the variables which control performance of multianchored walls. The primary factors which have been included in the analysis are: (1) the wall flexibility; (2) the anchor stiffness, (3) the embedment depth; (4) the initial stress conditions.

ANALYSIS PROCEDURE

In structural mechanics, the flexure equation forms the basis of the theoretical analysis of the behaviour of a flexible elastic element, but the relationship between lateral pressure and deflection must be specified before such an approach can be used to analyse the deflections, moments and stresses in a flexible wall.

An efficient discrete-element solution for a flexible wall on elasto-plastic foundation has been presented by Haliburton (1968) and we will use an analogous model. The wall is divided into equal increments and effects are concentrated at the increment points along the structure. This procedure allows freely discontinuous variation of flexural stiffness, transverse and axial loads, and elasto-plastic foundation support along the wall.

However, the earth pressure variation law is a nonlinear function both in respect to deflection and depth. To represent nonlinear soil response, we consider that this response may be described at some point along the structure by a nonlinear lateral earth pressure-structure deflection curve, as seen in fig. 1.

Kondner (1963) has shown that the nonlinear stress-strain curves for a number of soils could be approximated reasonably accurately by hyperbolas. If we substitute the earth pressure deflection (p-w) relationship from Fig. 1 by two hyperbola branches, the relationship can be represented two equations of the form:

$$\varepsilon p = \varepsilon p_o - \frac{w}{\dfrac{1}{E_{oa}} + \dfrac{w}{p_a}} \text{ (for } w < 0) \qquad (1\text{ a})$$

$$\varepsilon p = \varepsilon p_o + \frac{w}{\dfrac{1}{E_{op}} + \dfrac{w}{p_p}} \text{ (for } w > 0) \qquad (1\text{ b})$$

in which p_o = at rest pressure, p_a = active pressure, p_p = passive pressure, E_{oa} = initial tangent modulus of soil in expansion, E_{op} = initial tangent modulus of soil in compression and $\varepsilon = 1$. In the above equations the absolute values are considered for all parameters. The sign convention is given in fig. 1.

The relationships above apply to the soil mass behind the structure and above the dredge line. For the soil below dredge line, two different curves must be developed because passive pressure increase and active pressure decrease are ocurring at the same time on different sides of the structure. The right-hand curve corresponds to that shown in fig. 1, whereas the left-hand curve is a symmetrical one with respect to the origin, but with different values. The corresponding equations for the left-hand curve are given by (1a) and (1b), in which $\varepsilon = -1$ and p_a and E_{oa} values are interchanged with p_p and E_{op} values. The two curves are then automatically superimposed and a combined curve developed.

The deformation characteristics of the anchors can be introduced by a specified anchor yield value or, more realistically, by a specified anchor rod curve. As the force-deformation curve for anchor rods has a general form, it was considered more suitable to make provision that this curve be introduced by points.

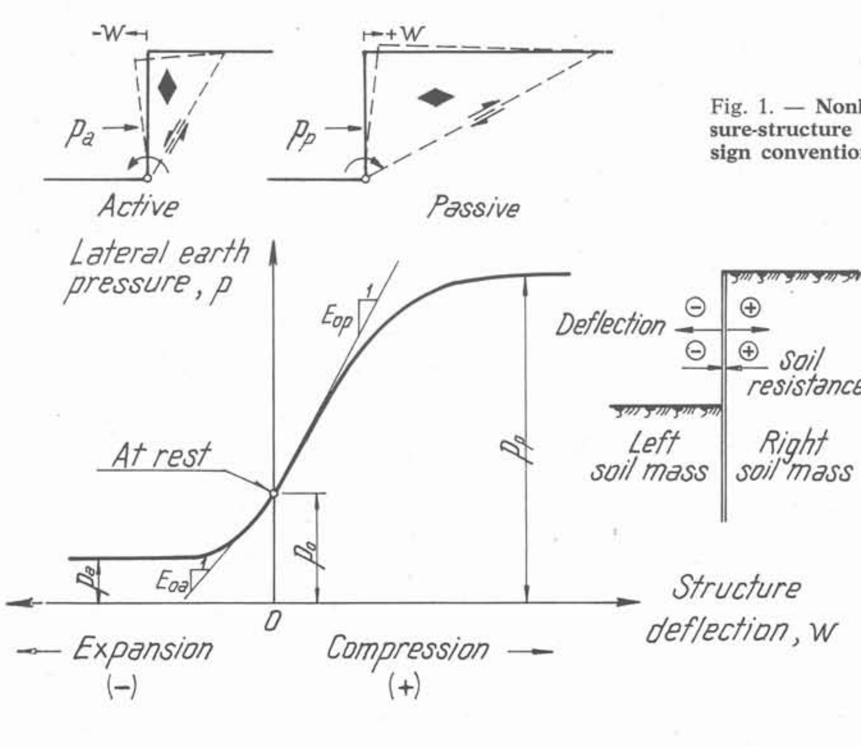

Fig. 1. — **Nonlinear lateral earth pressure-structure deflection curve and sign convention.**

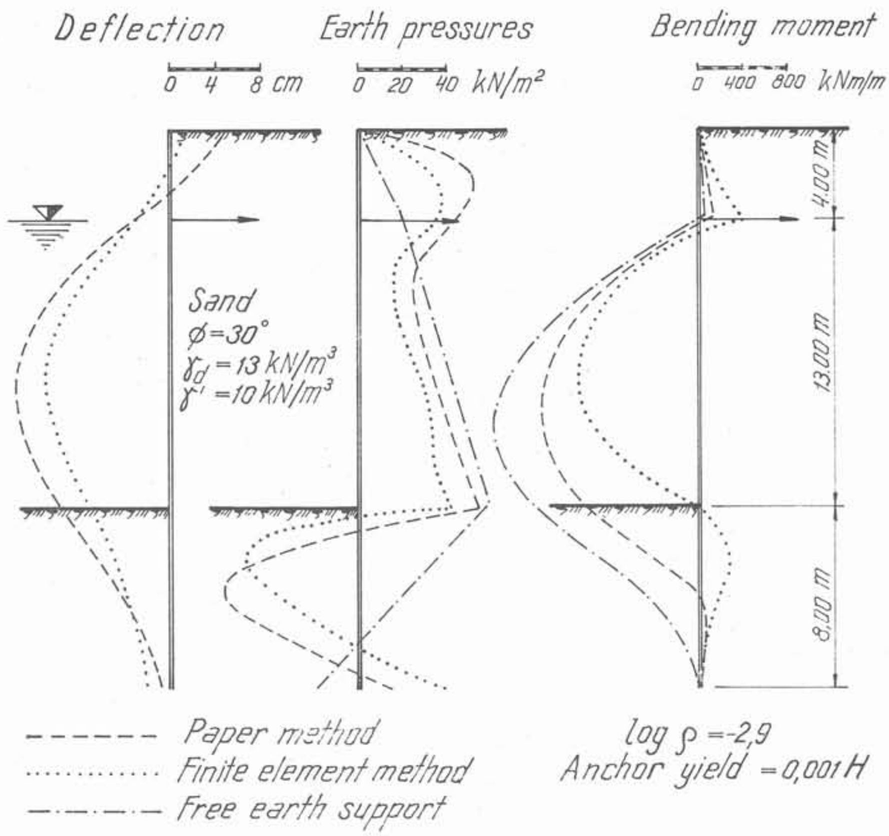

Fig. 2. — **Comparison of the proposed procedure results with the results obtained from other methods.**

To solve a flexible wall with nonlinear anchor-supports on nonlinear soil, repeated trial and adjustment solutions are made with a simple complete elastic wall solution in each trial. The convergence criterium consists of the comparison between two subsequent elastic lines of the wall. Consequently a digital computer solution was used.

To test the capabilities of applying this particular procedure to the anchored flexible wall problem, one of the walls included in Bjerrum's State of the Art Report at the Madrid Conference (1972) was analysed and the comparative results are presented in fig. 2. The computed deformations, earth pressure distribution and bending moments are in fairly good agreement with those obtained from the finite element analysis. The results obtained from the «free earth support method» are also given in fig. 2.

The differences between the results of the method proposed in the paper and the results obtained from the finite element analysis are small and on the safe side. Taking into account its simplicity and the saving of the computation time, the numerical procedure used in this paper seems to be a good tool for obtaining a proper solution for anchored flexible walls.

CHARACTERISTICS OF THE ANALYSED MULTIANCHORED WALLS

It has been established that there are a large number of variables that influence the behaviour of an anchor-supported flexible wall. Single anchor walls has been investigated in detail by Rowe (1955). By introducing a coefficient of flexibility

$$\rho = \frac{H^4}{EI} \tag{2}$$

in which H is the total height of the wall, E, its modulus of elasticity and I, its modulus of inertia, Rowe gave for the first time a quantitative correlation between the anchor reaction and the flexibility of the wall. The important effect of flexure below the dredge level and flexure above the anchor level, was pointed out.

In designing the multianchored walls, the empirical rules for multistrutted walls are often applied, despite the fact that the experience underlying the recommended rules is meagre. By assuming a trapezoid (Tschebotarioff, 1951) or a rectangular (Peck, 1969) earth pressure distribution, the spacing of the tie-back is adjusted so that the same load is transmitted in each tie-back (fig. 3). The load-redistribution effects of wall continuity and the effects of yielding of each anchor-support are disregarded.

In order to supplement the small amount of empirical data available, it was decided to attempt to analyse the anchor reaction variation in respect to some of the primary influence factors, by means of the proposed procedure.

This study is based on the analysis of a 14 m length wall, having a 4 m embedment depth. The wall was considered supported by two, three and respectively four anchor rows. The layout of the systems is shown in fig. 4. The soil, with a nonlinear behaviour has the following characteristics: no cohesion, angle of internal friction = 30°, unit weight = 19 kN/m³, initial tangent soil modulus in compression linearly increasing with depth, according to

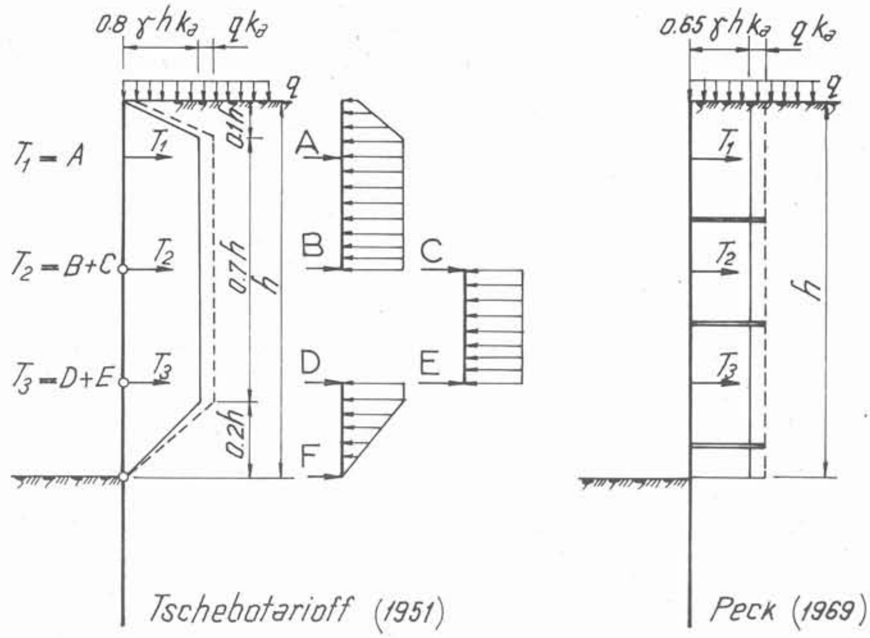

Fig. 3. — **Semi-empirical rules for anchor load prediction.**

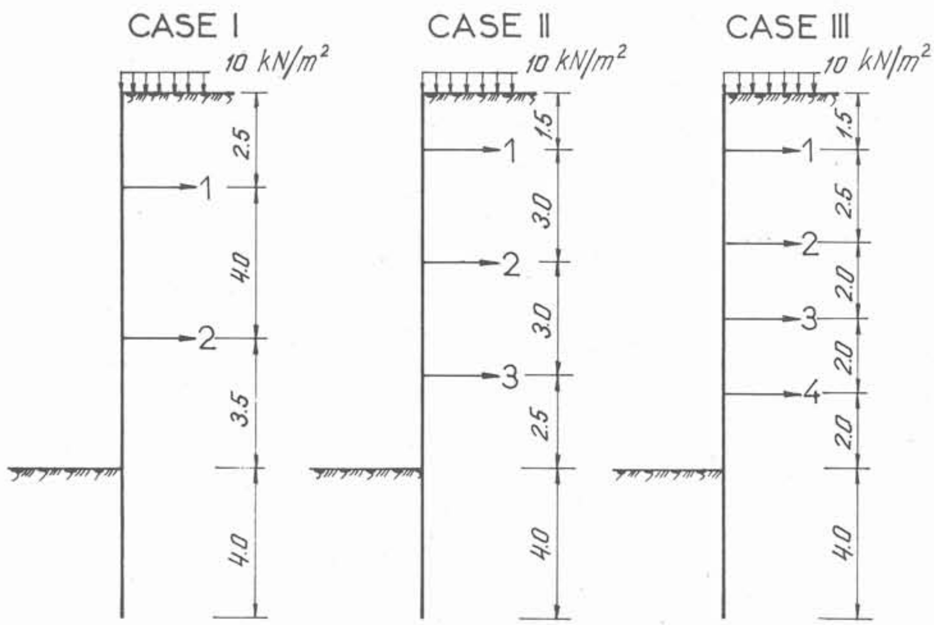

Fig. 4. — **Layout of the systems.**

Terzaghi (1955) expression:

$$E_{op} = l_h \, (z/D) \qquad (3)$$

where $l_h = 3\,000$ kN/m³, D is the effective embedment depth, z, the equivalent soil depth taking into account the surcharge load. It was assumed that $E_{oa} = 0.5\,E_{op}$.

Initial stresses in the soil are taken as those corresponding to an at-rest condition with a lateral pressure coefficient $K_o = 0.577$.

In order to analyse the influence of wall flexibility on anchor reaction a large range of wall flexural stiffness values were incorporated in the study, namely from 28 850 kN/m² to 1 920 000 kN/m². Actual walls with similar stiffness values would range from a 2B type Larsen sheetpile to a 1.0 m thick concrete slurry wall. The corresponding range of the Rowe flexibility coefficient, log ρ, is -2.83 to -4.65, ρ being introduced in Rowe's original units (ft⁴/lb in² per ft).

EFFECT OF WALL FLEXIBILITY AND ANCHOR STIFFNESS ON ANCHOR REACTION

Questions were raised concerning the wall stiffness influence on anchor reaction values. This influence can be analysed only in direct relation with the support stiffness.

The wall stiffness values considered in this study range from those corresponding to very flexible sheet pile walls to values corresponding to very stiff concrete cast-*in-situ* walls. Il is probable that for the slurry trench wall cracking can occur, resulting in a substantial reduction in wall rigidity. In such cases the cracked moment of inertia is recommended for use in calculations.

In order to study the influence of anchor yield, three values of this parameter were considered in the present study, namely 0.0, 0.1 and 0.2% of the wall height. The results of the analyses are presented in fig. 5 a, b and c, showing the calculated anchor reaction values versus wall flexibility for different anchor yields. The diagrams are self-explanatory and it may be observed that the anchor reaction values depend in a great degree on the wall flexibility an anchor yield.

For the zero anchor yield case, the lower tie-back is the most loaded and the reaction difference between this anchor and the upper anchors is very large. The more flexible the wall, the less low the anchor reaction.

As expected, the greater anchor yield results in a decrease of the lower anchor reaction. Also it results in a decrease of the upper anchor reaction, for the usual range of wall flexibility. For 0.1% and 0.2% anchor yield cases, the wall flexibility increase yields to an increase of the lower support reaction and a decrease of the upper support reaction.

If the double-anchored wall is analysed it may be observed that the two anchors are equally loaded for a particular value of wall flexibility. This particular value increases with anchor yield increase. For the walls with three and four anchor rows, there is no particular value of wall stiffness resulting in an equal load in each individual anchor.

As regards the anchor loads predicted by means of the semi-empirical rules treated by Tschebotarioff and Peck, they overestimate the upper anchor pull and understimate the lower anchor pull.

The presence of tie-backs can be more realistically considered by introducing in the support points the corresponding nonlinear anchor deformation curves rather than an arbitrary anchor yield.

Tie-backs can very in stiffness by a factor of 10 depending upon whether tie rods or multistrand cables are used. In these analyses two values of anchor stiffness, EA, were considered namely 3.5 . 10³ kN/m and 3.5 . 10⁴ kN/m. The anchor nonlinear curve characteristics are based on the elasto-plastic behaviour of steel.

The effect of anchor stiffness on anchor reactions is shown in fig. 6 for all three analysed cases. The main outcome of the results presented in fig. 6 is that the stiff anchors are more unequally loaded than the flexible anchors.

When stiff tie-backs are used, the lower anchor is the most loaded one, whereas the upper anchor is the least loaded one, under all circumstances. The difference between anchor pull values may be as high as 80% for the flexible walls.

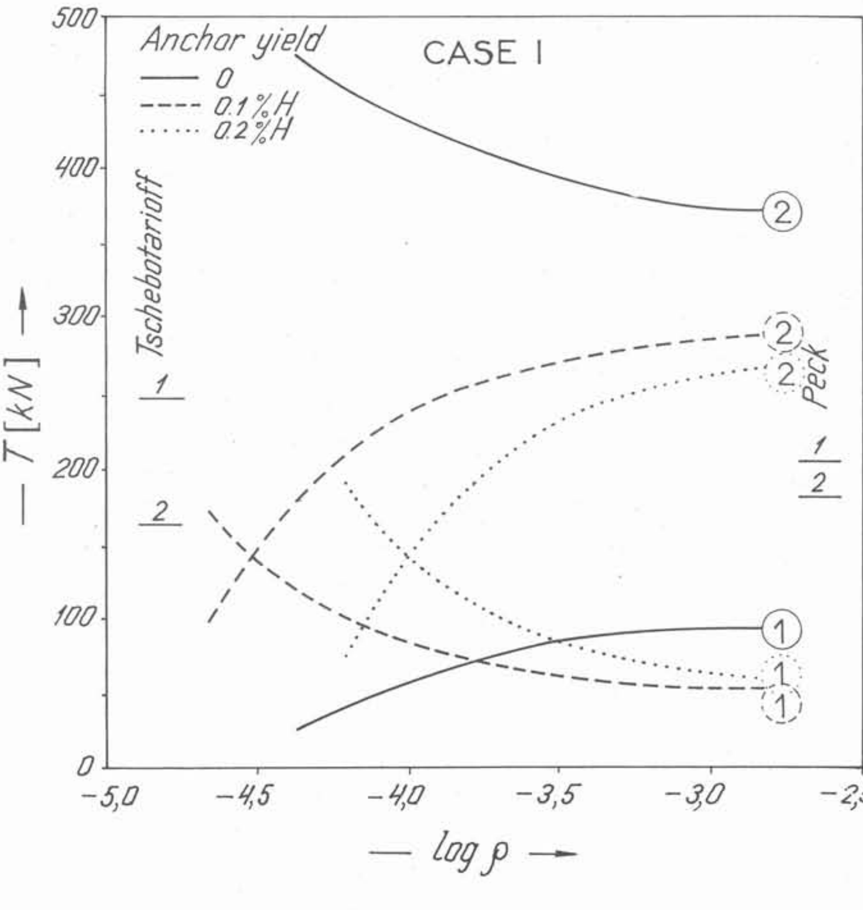

Fig. 5 a. — **Anchor reaction versus wall flexibility and anchor yield (case I).**

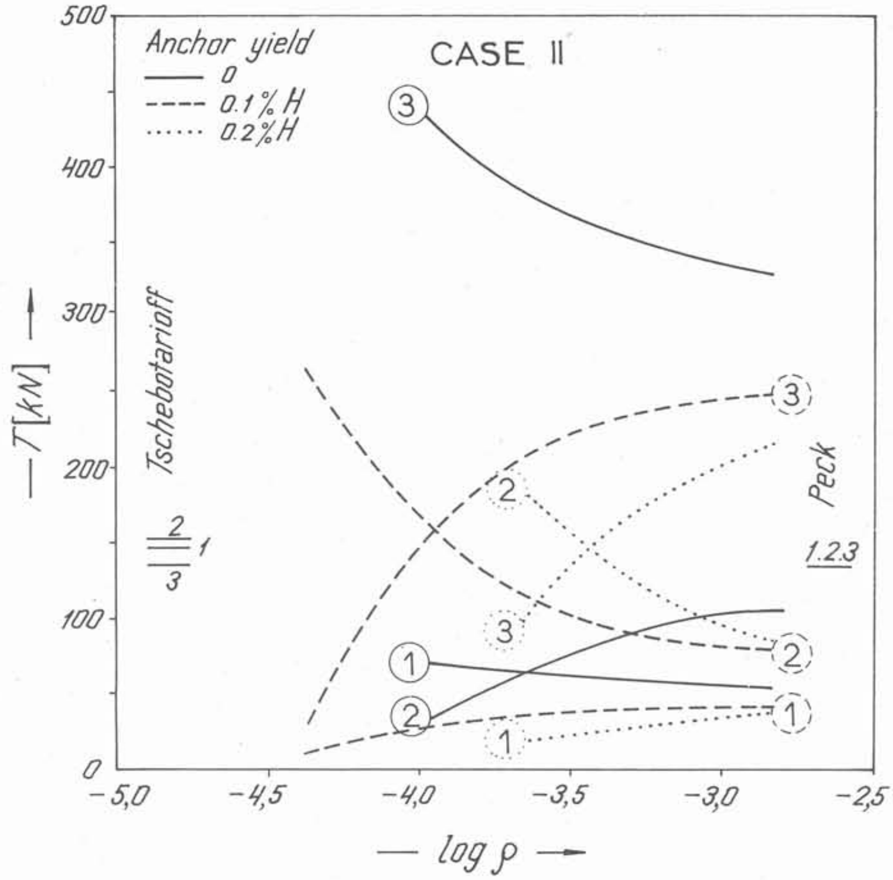

Fig. 5 b. — **Anchor reaction versus wall flexibility and anchor yield (case II).**

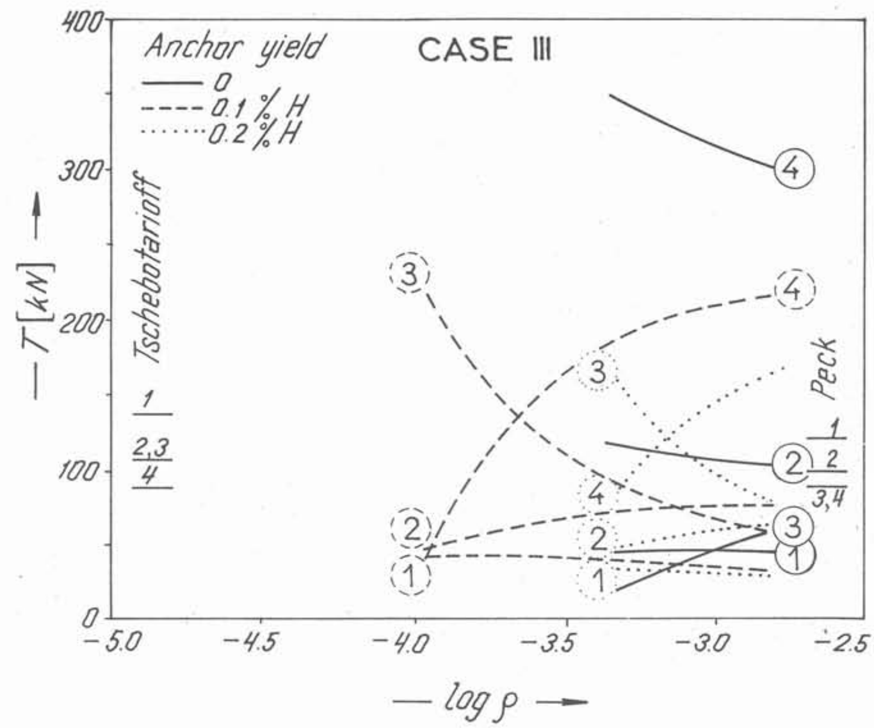

Fig. 5 c. — **Anchor reaction versus wall flexibility and anchor yield (case III).**

When flexible tie-backs are used, they are more uniformly loaded. For each case analysed, there is a particular wall stiffness value when the reactions of individual anchors are practically equal. This particular value corresponds to a log ρ of -3.75, for the double-anchored wall and -4.00, for the walls supported by three and four anchor rows.

In general, as the wall flexibility increases, the upper anchor load decreases and the lower anchor load grows, for both anchor stiffness values. For the intermediate anchors, the anchor pull shows an increasing or a decreasing tendency, but these variations are not quite important. It may be also noted that the stiff anchor load is less influenced by the wall flexibility, when comparing with the flexible anchors.

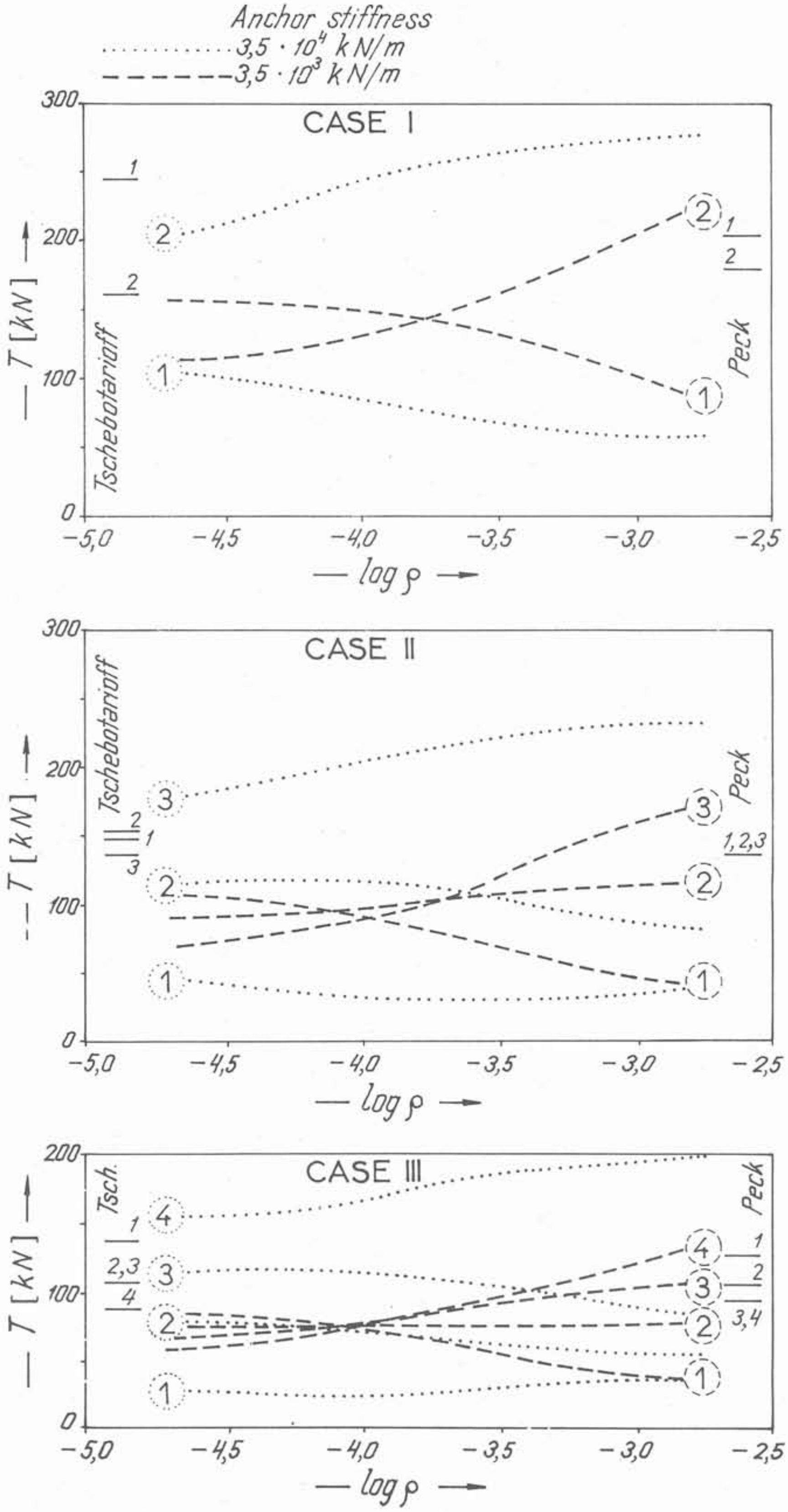

Fig. 6. — **Anchor reaction versus wall flexibility and anchor stiffness.**

The computed anchor yields are (0.0055 — 0.055) %H for the stiff tie-backs and (0.065 — 0.45) %H for the flexible tie-backs. It can easily be seen that a zero deflection (simple support) condition at each anchor location would have been too restrictive. The stiffer tie-backs reduce wall deflections, but the reduction is not in proportion to the stiffness change, since an increase in tie-back stifness by a factor of 10 is required to cause a 30-60% reduction in movements.

Comparison of the computed anchor reactions with those estimated by means of the semi-empirical rules proposed by Tschebotarioff and Peck, again shows differences, which grow with wall stiffness decrease and anchor stiffness increase. As can be seen from fig. 6 these differences are not always on the safe side.

EFFECT OF THE INITIAL STRESS CONDITIONS AND EMBEDMENT DEPTH ON ANCHOR REACTION

In order to begin our computations it is necessary to define the initial horizontal stresses which are assumed to correspond to an at-rest condition.

In the determination of earth pressure at rest, there constantly occur discrepancies between experimental and theoretical results. There were several authors who underlook research on the determination of the coefficient of earth pressure at rest for granular soils, and presented different equations (Myslivec, 1972). Using these equations with our soil characretistics, the following values of the coefficient of at rest pressure were obtained: Jaky: $K_o = 1 - \sin \varnothing = 0.5$; Brooker and Ireland: $K_o = 0.95\,(1 - \sin \varnothing) = 0.475$; de Wet: $K_o = (1 - \sin^2 \varnothing)/(1 + \sin^2 \varnothing) = 0.5$; Siedek: $K_o = 0.75\ \mathrm{tg}^2\,(\pi/4 - \varnothing/2) + 0.25 = 0.499$; Wierzbicki: $K_o = \mathrm{tg}^2\,(\pi/4 - \varnothing/3) = 0.491$.

Pruška (1972) showed that the conditions for the so-called pressure at rest are fulfilled for different values of vertical to horizontal pressure ratio, which form a conical zone, I (fig. 7). The lower and the upper limits of this zone are defined by the equations:

$$K_{o1} = \mathrm{tg}\,(\pi/4 - \varnothing/2) \qquad (4\ a)$$

$$K_{o2} = \mathrm{tg}\,(\pi/4 + \varnothing/2) \qquad (4\ b)$$

The zones II are the local plastic area zones and the zones III are impossible strength zones. For our particular soil $K_{o1} = 0.577$ and $K_{o2} = 1.732$.

It may be observed that Jaky's equation as well as the other previous equations, refer to the lower limit of the coefficient of pressure at rest.

The influence of the coefficient of earth pressure at rest and the influence of embedment depth on the anchor reactions, for the double-anchored wall, are illustrated in fig. 8. The anchors stiffness is $3.5 \,.\, 10^3$ kN/m.

As embedment depth increases the lower anchor reaction decreases, tending to a constant value, whereas the upper anchor reaction increases. For the very stiff wall, the upper reaction reaches a maximum value and then slightly decreases when embedment depth increases. The most important variations (40 — 100%) of the anchor reaction values occur in the embedment depth range 2 to 4 m. It is to be noted that below the 4 m embedment depth the values of the tie-back reactions, so far constant, quickly differentiate, thus making the upper tie-back completely useless. As far as the lower one is concerned, this reaches the over high values given in fig. 8. The corresponding soil restraint is of a sinking support type.

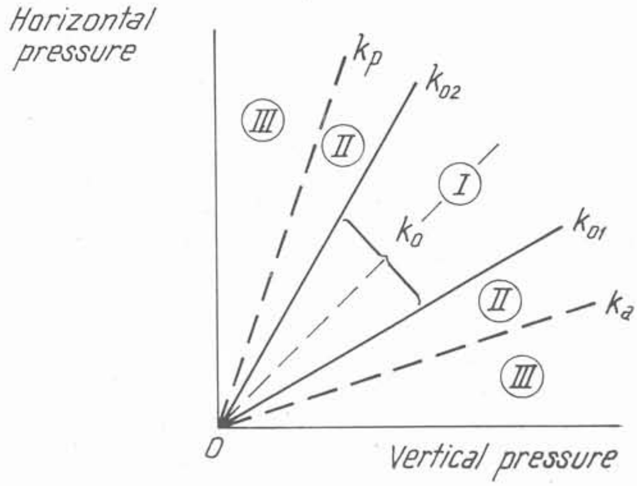

Fig. 7. — **Limits of the earth pressure at rest zone.**

The effect of K_o increase is a reduction of the lower anchor reaction. During the same time, the upper anchor load increases, for the very flexible wall and decreases, for the very stiff wall. But the anchor load changes induced by the K_o variation between the two extreme limits, are not greater than 15% and are relatively independent of embedment depth.

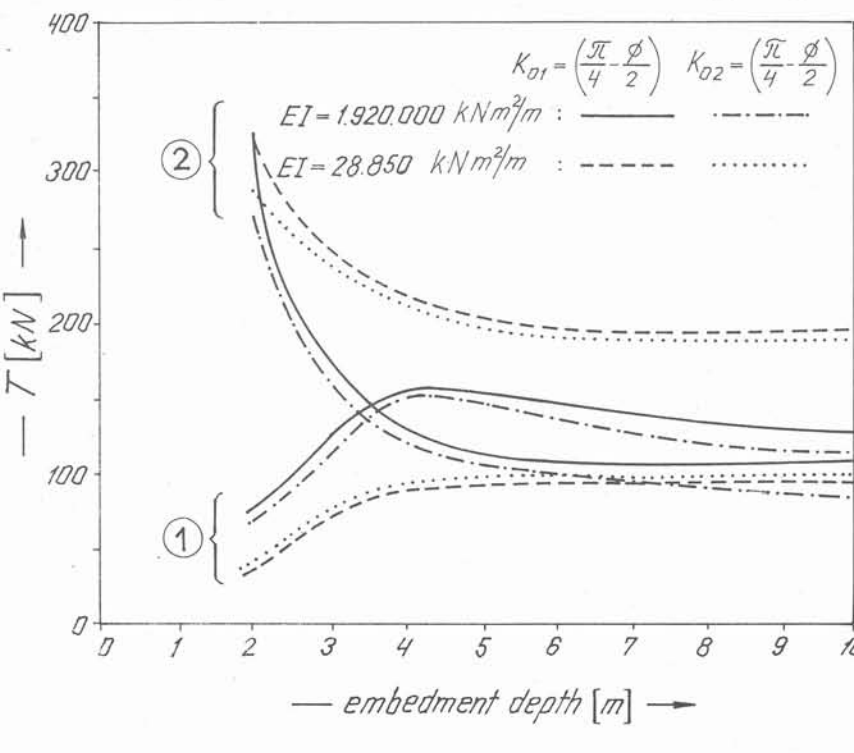

Fig. 8. — **Anchor reaction versus coefficient of earth pressure at rest and embedment depth.**

CONCLUSIONS

The design of a tied-back flexible wall is a complex operation which depends on knowledge of the interaction between the anchors, the ground and the wall.

The numerical procedure for computer analysis of the anchored walls used in this paper takes this interaction into consideration. Nonlinear anchor behaviour and nonlinear soil reponse can be properly represented. Capabilities of the method have been illustrated by the good agreement between the method results and the finite element analysis results. It should be noted however that the accuracy of the method can obviously not be better than that of the input-data provided.

The primary purpose of this study was to examine, by a numerical method, how the anchor reaction is influenced by some of the primary factors which control the over-all behaviour of multianchored flexible walls. This objective was achieved by considering the influence of the wall flexibility, anchor stiffness, embedment depth and initial stress conditions on anchor pull values.

In the design of tied-back wall systems the anchor pull evaluation is often based upon the semi-empirical earth pressure diagrams such as those suggested by Tschebotarioff and Peck. The comparison of the values obtained by these practical rules with the results of our analyses shows that the differences are not always on the safe side for the anchor forces evaluated by means of these rules.

It may be concluded that the computation method must be chosen on the basis of the extent to which it simulates the actual behaviour of real anchor-ground-wall system. Thereafter the value of the safety factor in respect to the ultimate pull-out capacity of the individual anchor and its application might well be left in the hands of the designer. This factor of safety allows for time effects, ground variability, repetitive loading and grouping action.

Despite the limitations related to the particular soil and structures considered in this analysis, it is believed that the findings of the paper are of general value for engineering evaluation of the over-all problem of the anchor systems behaviour.

REFERENCES

[1] BJERRUM (L.) et al. — «Earth pressure on flexible structures - A State of the Art Report», *Proc. 5th European Conf. SMFE,* vol. 1, Madrid, pp. 169-196 (1972).

[2] CLOUGH (G.W.) and TSUI (Y.). — «Performance of tied-back walls in clay», *Journ. Geotechn. Div.,* vol. 100, no GT 12, pp. 1259-1273 (1974.)

[3] HABIB (P.). — «Les ancrages, notamment en terrain meuble», Rapport Général, Sess. Spécial 15, Proc. 7th ICSMFE, Mexico (1969).

[4] HALIBURTON (T.A.). — «Numerical analysis of flexible retaining structures», *Journ. SMFD,* vol. 94, no SM 6, pp. 1233-1251 (1968).

[5] KONDNER (R.L.). — «Hyperbolic stress-strain response: cohesive soils», *Journ. SMFD,* vol. 89, no SM 1, pp. 115-143 (1963).

[6] MYSLIVEC (A.). — «Pressure at rest of cohesive soils», *Proc. 5th European Conf. SMFE,* Madrid, vol. 1, pp. 63-67 (1972).

[7] PECK (R.B.). — «Deep excavations and tunneling in soft ground», *State of the Art Volume,* 7th ICSMFE, Mexico, pp. 225-290 (1969).

[8] POPESCU (M.). — «A numerical analysis of flexible cast-*in-situ* diaphragm walls considering nonlinear soil behaviour», *Constructions Review,* no 5, pp. 17-22 (1977).

[9] PRUŠKA (L.). — «Basic equations of pressure at rest of granular materials», *Proc. 5th European Conf. SMFE,* Madrid, vol. 1, pp. 69-76 (1972).

[10] ROWE (P.W.). — «A theoretical and experimental analysis of sheet-pile walls», *Proc. Inst. of Civ. Eng.,* London, part 1, pp. 32-69 (1955).

[11] TERZAGHI (K.). — «Evaluation of coefficients of subgrade reaction», *Geotechnique,* vol. 5, pp. 297-326. (1955).

[12] TSCHEBOTARIOFF (G.P.). — «Soil Mechanics, Foundations and Earth Structures», Mc. Grow-Hill, N.Y. (1951).